

# Upgrading Relations List in Fuzzy Cognitive Maps Using Reinforcement Learning

Frank Balmaseda, Mabel Frias, Frank Verstappen

Hasselt Universiteit Martelarenlaan,  
Belgium

{frank.balmaseda, mabel.friasdominguez,  
frank.verstappen}@uhasselt.be

**Abstract.** Fuzzy Cognitive Maps (FCM) are dedicated to modeling complex dynamic systems and has been widely studied. One of those studies probed that Computing with Words (CWW) is very effective to improve the interpretability and transparency of FCM. Learning methods to calculate the weight matrix in a map are the target of hundreds of studies in various parts of the world. These methods allow the map to learn or evolve towards a better state, but always taking into account that the output must be evaluated and compared using a real scenario. Reinforcement Learning has, within its performance, one of the possible answers to this problem, aimed at improving the classification capacity of the map and thus improving the learning of its relations list. This paper presents a new learning method for Fuzzy Cognitive Maps. The proposal was evaluated using international databases and the experimental results show a satisfactory performance.

**Keywords:** Fuzzy cognitive maps, reinforcement learning, relations list.

## 1 Introduction

Cognitive maps were presented for the first time in 1976 by [3], their main objective being to represent social scientific knowledge through designated digraphs where the arcs are causal connections between the nodes. Fuzzy Cognitive Maps (FCM) [19], were introduced as an extension of Cognitive Maps theory, and they are recurrent structures modeled through weighted graphs.

In this representation, each node or concept of the graph may represent a variable, an object, entity or state of the system that is intended to be modeled; while the weights in the connections determine the causality between these concepts.

There are two fundamental strategies to build FCMs: manual and automatic. In the first variant, the experts determine the concepts that describe the system, as well as the direction and intensity of the causal connections between the neurons.

In the second variant, the topology of the map and the weight matrix are constructed from historical data, or representative examples that define the behavior of the system. Regardless of the strategy, domain experts need to define the architecture that best fits the system that is intended to model, as well as the restrictions inherent in the model [21].

The formulation of learning algorithms to estimate the causality of the system (matrix of causal weights that regulate the interaction between the concepts) is still an open problem for the scientific community [23]. The learning algorithms that are dedicated to the classification of the weights in FCM still present gaps, both in the efficiency and in the results that they offer.

For instance, in [5] authors recognize that using Ant Colony System outperforms RCGA, NHL and DD-NHL algorithms in terms of model error, but only when multiple response sequences are used in the learning process, not so when one response sequence is used.

Other studies like in [17] and [26], shows the proposals are a little bit slow because for computing every weight it is necessary to consider the other concepts involved in the causal-effect relation for a target concept. The broad attention of the authors to the subject reinforces the premise of its importance.

Within this framework, there is a clear need to improve the methods used (or, as is the case of this paper, to create new ones) up to now to carry out the learning of the FCM.

## 2 Fuzzy Cognitive Maps and Computing with Words

In a FCM the inference can be defined mathematically using a state vector and a causality matrix. The state vector  $A_{1 \times N}$  represents the activation degree of the concepts, and the causal weight matrix  $W_{N \times N}$  defines the interaction between the concepts.

The activation of  $C_i$  will depend on the activation of the neurons that directly affect the concept  $C_i$  and the causal relationships associated with that concept. The process of inference in an FCM is then summarized in finding the value of the state vector  $A$  through time for an initial condition  $A^0$  as can be seen in the equation (1):

$$A^{(t+1)} = f\left(\sum_{j=1}^N A^{(t)} W_{ji}\right), i = j \quad (1)$$

The causal relationships between the concepts can occur in 3 ways. For two concepts  $C_i$  and  $C_j$  it is fulfilled that:

- $W_{ij} > 0$ : Indicates a positive causality between the concepts  $C_i$  and  $C_j$ , which means an increase (decrease) in the value of  $C_i$  leads to the increase (decrease) in the value of  $C_j$ .
- $W_{ij} < 0$ : Indicates a negative causality between the concepts  $C_i$  and  $C_j$ , that is, the increase (decrease) in the value of  $C_i$  leads the decrease (increase) in the value of  $C_j$  proportional to the absolute value of  $W_{ij}$ .
- $W_{ij} = 0$ : Indicates the non-existence of a causal relationship between  $C_i$  and  $C_j$ . It can also occurs when  $W_{ij}$  is very close to zero.

On the other hand, CWW [30] was presented as a methodology that allows introducing linguistic variables with the objective of performing computational operations with words instead numbers. Linguistic variables describe situations that cannot be clearly defined in quantitative terms and allow a very particular type of

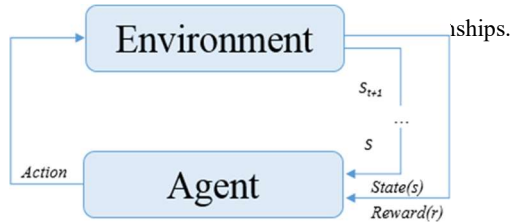


Fig. 1. Behavior of a reinforcement learning problem.

translation from natural language to numerical or numerical sentences. Some models inspired in his technique are briefly described below:

- *Linguistic Computational Model based on membership functions*. The linguistic terms are expressed by fuzzy numbers, which are usually described by membership functions. This computational model makes the computations directly on the membership functions of the linguistic terms by using the Extension Principle [9].
- *Linguistic Computational Symbolic Model* [6]. This model performs the computation of indexes attached to linguistic terms. Usually, it imposes a linear order to the set of linguistic terms  $S = S_0, \dots, S_g$  where  $S_i < S_j$  if and only if  $i < j$ .
- *The 2-tuple Fuzzy Linguistic Representation Model* [15]. The above models perform simple operations with high transparency, but they have a common drawback: the loss of information caused by the need of expressing results in a discrete domain. The 2-tuple model is based on the notion of symbolic translation that allows expressing a domain of linguistic expressions as a continuous universe.

### 3 FCM and Reinforcement Learning

The main problem of reinforcement learning methods is to find an optimal policy of actions that is capable of reaching a goal maximizing the rewards. Starting from an initial state, the agent chooses an action in each iteration, which is evaluated in the environment and receives penalties or rewards according to the results [27].

The Figure 1 shows a general performance of this technique.

The learning methods for FCM in the literature formulate several hypotheses and most of them have application results that improve the classification of FCM. Below, some of these approaches are briefly described:

- *Differential Hebbian Learning* [8] is based on the following principle: if the cause concept  $C_i$  and the effect concept  $C_j$  change the activation value simultaneously, then the causal weight  $W_{ij}$  will increase with a constant factor, otherwise the causality will not be modified in that iteration, see equation 2:

$$w_{ij}^{(t+1)} = \begin{cases} w_{ij}^{(t)} + \gamma^{(t)}[\Delta A_i / \Delta A_j - w_{ij}^{(t)}] & \text{if } \Delta A_i \neq 0 \\ w_{ij}^{(t)} & \text{if } \Delta A_i = 0 \end{cases} \quad (2)$$

- (*Balanced Differential Algorithm* [17]) proposes to use the activation degree of the nodes that are simultaneously modified during the weight matrix updating. This algorithm assume it is a FCM without concepts using auto-connections and that it is adjusted iteratively.
- In [20] the authors propose a method based on Evolutionary Strategy to adjust the structure of the map using historical data, where each instance is composed of a pair of vectors. The first vector encodes the activation degree of the input concepts, while the second vector denotes the response of the system for the specified configuration.
- In [24] the authors applied Particle Swarm Optimization to estimate the appropriate structure of the system from a sequence of multiple state vectors. The peculiarity of this method lies in its learning scheme: the response of the system is defined by decision-making nodes.
- A new automated approach based on Genetic Algorithms with Real Coding can be found in [26]. The idea of the method is to maximize the objective function  $f(x) = 1/(1 + \alpha(x))$ , reducing the global error of the map.
- In [5] the authors presented a novel algorithm inspired in Ant Colony Optimization to adjust maps with dozens of concepts. In this model, each weight is encoded as a sequence of discrete states. During the search process, each ant constructs a solution. In the next step, each discrete solution is transformed to its continuous equivalent, resulting in a matrix of causal weights.

### 3.1 Learning Algorithm

The method proposed in [10] uses a Learning Automata (LA) [22, 28], which are simple reinforcement learning components for adaptive decision making in unknown environments. An LA operates in a feedback loop with its environment and receives feedback (reward or punishment) for the actions taken. The general update scheme is given by 3 and 4:

$$p_m(t+1) = p_m(t) + \alpha_{reward}(1 - \beta(t))(1 - p_m(t)) - \alpha_{penalty}\beta(t)p_m(t), \quad (3)$$

if  $a_m$  is the action taken at time  $t$ :

$$p_j = p_j(t+1) - \alpha_{reward}(1 - \beta(t))p_j(t) + \alpha_{penalty}\beta(t)(r-1)^{-1} - p_j(t), \quad (4)$$

if  $a_m \neq a_j$

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**Algorithm 1** Establishing classification

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- 1: Create map (concepts and relationships). Executed by an expert
  - 2: Assign matrix weights to the map using FCM+CWW+RL. Executed by expert
  - 3: Assign to each concept  $C_i$ , the value of each feature of the object to be classified.
  - 4: Assign the linguistic term "NA" to each decision concept.
  - 5: Calculate the activation value of the concepts
  - 6: Classify a new object  $O_c$  using equation 5.
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**Algorithm 2** FCM+CWW+RL

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- 1:  $\forall i = 1...n, \forall j = 1...n, M_{ij} = 1/n$
  - T2: Obtain reward  $r_i =$
  - 3: Update  $M$  using  $r_i$ .
  - 4: Repeat from step two until reach the stopping condition
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where  $p_i(t)$  is the probability of selecting action  $i$  at time step  $t$ . The constant  $\alpha_{reward}$  and  $\alpha_{penalty}$  are the reward and penalty parameters. When  $\alpha_{reward} = \alpha_{penalty}$ , the algorithm is referred to as linear reward-penalty ( $L_{R-P}$ ), when  $\alpha_{penalty} = 0$ , it is referred to as linear reward-inaction ( $L_{R-I}$ ) and when  $\alpha_{penalty}$  is small compared to  $\alpha_{reward}$ , it is called linear reward  $\epsilon$  penalty ( $L_{R-\epsilon P}$ ).  $\beta(t)$  is the reward received by the reinforcement signal for an action taken at time step  $t$ .  $r$  is the number of actions [29].

In [10] authors propose the use of RL in decision making problems specifically in Personnel Selection. Basing in this approach, in this paper it is proposed using RL as a learning method to adjust the weight matrix of a FCM+CWW. To reach this objective define a FCM+CWW as the environment over which the agent must make the decisions. The parameters to be used in the environment are defined as follows:

- Firstly, the size of the list of terms that are used in the map: For this proposal a study is included in [11] that states, through a transfer function, the use of linguistic terms instead numerical methods traditionally used to represent the weights in the causal relationships of the maps. This list may vary depending on the needs of precision in the domain, therefore, it can be simplified to three terms (Low, Medium, High), however, the author suggests to use fifteen terms as described in [11]
- Length of the relations list: A FCM contains a list of relationships used to determine the connections between each concept involved in the map.
- Training set: These instances are necessary for the classification algorithm to train with that set and to return the best individual. The training set is created from the cross validation technique used to perform the data validation.

From this point, the environment continues with obtaining the reward and updating the weight matrix, as is shown in algorithm 1:

$$O_c = F(\arg \max_{A_k \in \mathcal{A}^D} \{A_k\}), \quad (5)$$

**Table 1.** Matrix built from the list of relationships.

<b>R1</b>	<b>R2</b>	<b>R3</b>	<b>R4</b>	<b>R5</b>
$R_{i_1}$	$R_{i_2}$	$R_{i_3}$	$R_{i_4}$	$R_{i_N}$
$R_{j_1}$	$R_{j_2}$	$R_{j_3}$	$R_{j_4}$	$R_{j_N}$
W1	W2	W3	W4	W5

where  $F$  returns the value of the class  $D_k$  and  $A^D$  is the activation set of the decision concepts. It is worth mentioning that our symbolic model based on FCM conserves its recurrent nature.

This implies that the FCM will produce a state vector composed by linguistic terms in each iteration until it discover a fixed point or reach a maximum number of iterations. Next step will be building a  $N \times 4$  matrix using the relations list. In Table 1 is shown a generic matrix of the relations list which describes the behavior of each relation. Here  $R1$  is a relation,  $R_{i_1}$  is the starting concept of  $R1$  and  $R_{j_1}$  is the target concept of  $R1$ . In this approach, the main idea is not to add relations to the list, but using the initial list of relations, change the  $R_i$  and  $R_j$  and keeping the same weight, which will be estimated using [4].

This method suggests using the classification accuracy value as a reward. This value tends to 1 when the reward is high. In equation (6) we formalize the expression used to calculate the reward once the classification is done, where  $r_i$  is the reward in the iteration,  $\delta_i$  is the number of well-classified objects for the same iteration, and  $T$  is the total instances of the data set:

$$r = \frac{\delta_i}{i_T}. \quad (6)$$

With this value, the agent performs the updating of the weight matrix and the reward is calculated in each iteration, which substitutes the time  $T$  commonly used for this type of problem. Below, the steps of the method are detailed presented in algorithm 2.

Modifying the reward function on each iteration allows finding a new state of the map and evaluating it in order to determine improvements in the classification, otherwise, this state will not be visited again through the iterations.

## 4 Experimental Results

This section presents the experimental framework that uses 20 data sets from [2]. The cross-validation method [7] was used to validate the results that subdivided the data set into  $k$  subsets of equal size (in this experimental frame  $k = 10$ ), of which one part makes up the set of tests and the other part the set of training. When using this technique, the number of subsets with which we worked was taken into account, because with higher values for  $k$ , the trend of the real error range of the estimate is reduced.

**Table 2.** Results of accuracy for each method.

Data-sets	MLP	NB	SVM	RF	ID3	FCM+RL
apendicitis	82,09	85,09	86,49	80,26	70	87,41
blood transfusion	75,81	75,81	56,25	62	75,14	76,03
chocardiogramme	90,02	90,82	90,69	65,83	85,43	91,21
heart-5an-nn	72,95	81,8	82	74,2	65,55	83,03
iris	94,67	94,67	82,22	70,35	89,33	96,66
liver disorders	53,59	62,88	65,38	66	49,82	66,13
planing-relax	63,27	60,94	73,22	58,75	53,27	73,37
saheart	67,11	67,97	87,18	63,99	49,57	67,99
yeast3	91,3	91,29	93,34	92,36	88,8	93,42
pima 5an-nn	67,03	74,22	75,78	74,91	59,51	76,09
acute inflammation	100	100	100	99,87	100	100
Yield	51,78	62,5	65,34	63,8	53,57	66,48
phoneme	81,1	76,62	82	79,77	80,6	83,07
ecolio	81,87	68,51	81,6	64,88	62,5	82,01
iris-5an-nn	96,42	95,33	96	64,88	62,5	82,01
vehicle	68,72	54,48	70,68	59,32	58,75	70,93
wine	96,81	94,97	97	92,12	91,54	96,8
glasso	95,56	94,28	95,75	89,41	96,3	96,46
vertebra-column-3c	82,54	66,24	83,95	74	65,83	84,86
car	72,83	66,27	74,99	71,38	63,99	77,83

**Table 3.** Holm test for  $\alpha=0.05$  for classification accuracy, taking as control FCM+CWW+RL.

i	Algorithms	$z = \frac{(R^0 - R_i)}{SE}$	p	Holm Null Hypothesis
5	ID3	4.60014	0.000033	0.0004 Rejected
4	RF	4.35842	0.005008	0.01 Rejected
3	SVM	3.11039	0.005201	0.01 Rejected
2	MLP	2.62014	0.008776	0.025 Rejected
1	NP	1.71063	0.010283	0.05 Rejected

In addition, the estimate will be more precise, the variance of the error range real is greater and the necessary computation time also increases as the number of experiments to be performed increases. For the statistical analysis were used the hypothesis test techniques in [25, 13], Friedman and Iman-Davenport for multiple comparisons [18] in order to detect statistically significant differences in a group of the results.

Holm test [16] is used in these experimental studies to find significantly higher algorithms. These tests are suggested in [7, 14, 13, 12], where the authors agree in the relevance of using them for validating results on the reinforcement learning area. KEEL module Non-Parametric Statistical Analysis was also used in this work for the statistical processing of the experimental results [1].

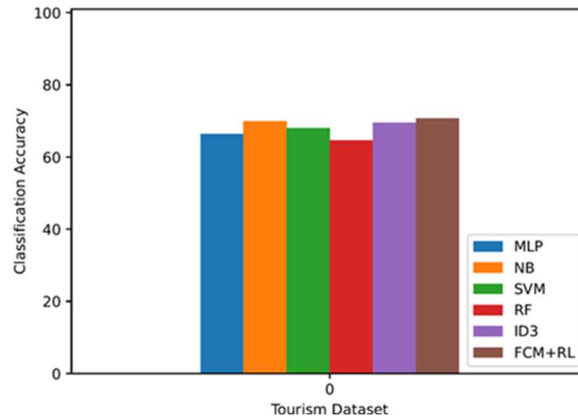


Fig. 2. Behavior of a reinforcement learning problem.

**Experiment 1:** Compare the method proposed (FCM+CWW+RL) with 3 algorithms: MLP, NB and ID3 which are implemented in WEKA tool.

**Goals:** Determine if the accuracy of the propose method FCM+CWW+RL is significantly superior to accuracy of MLP, ID3 and NB. In tables 2 and 3 are shown these results. Table 2 shows the list of all datasets used to compare the accuracy of this proposal with a well-known classification algorithm.

In table 3 it can be observed the results of applying the Holm test, and how the null hypothesis is rejected for all the algorithms with which this proposal is compared to.

## 5 Real Case Study

In tourism, we can find many challenges, but there is one that represents a continuous race to success. This challenge is to build the best network of stake- holders to keep services continuously at the same level. Suppliers vary according to the needs of the tourism facility (proximity, better products, fresh fish, fresh vegetables, etc.).

In the city of Camagey, Cuba, all hotels have been trying for a long time to find the best suppliers for their services. From an analysis of data and scenarios about these processes, we decided to collect data from suppliers and hotels to build a FCM and test the method proposed in this paper. Figure 2 details the results of the method proposed in this research using stakeholder and hotel data. As can be seen, the accuracy of our method outperforms the rest of the known algorithms selected for comparison.

## 6 Conclusions

In this paper, we have studied a new learning method that improves the classification in the FCM where the activation values of the concepts and the causal weights are described by means of linguistic terms. The results obtained show the effectiveness of using RL as a learning method to adjust the relations matrix in the FCM. According to the results presented in this paper, the approach is statistically better than MLP, NB, SVM, RF and ID3 in in terms of classification accuracy. Further research may include



performing hybridization with the method proposed in this research and that of [4] although many variations are possible.

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