

# Multi-Objective Product Allocation Model in Warehouses

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**Abstract.** The allocation of the merchandise to the different spaces available in the warehouses is a determining factor of the cost and the time of operation in said facilities, which is why it is important to establish the optimal storage positions, seeking better conditions of profitability and service for the Business. This article presents a multi-objective mathematical model to determine the allocation of the merchandise in the different spaces available in the warehouses, which simultaneously evaluates the operating costs and the times required to carry out the storage activities. The model is solved using a genetic algorithm, which due to the multiobjective nature, allows obtaining a set of possible accommodations of the merchandise, which can be used by the warehouse manager, sure that it will use an adequate allocation according to its preference in relation to the variables analyzed, which constitute an optimal relationship between the cost and the time required for storage.

**Keywords:** warehousing, merchandise allocation, multi-objective model, genetic algorithm, optimization.

## 1 Introduction

Products allocation is one of the key activities in warehouses optimization, since the appropriate selection of the places where the merchandise must be stored is a key activity to reduce operative costs and time. It is possible due to the reduction of the distances and the efforts required moving the goods inside these facilities, which allows reducing the costs of material handling and labor hours, as well as the time required for internal operations and thus the time of order fulfillment.

In the specialized scientific literature, there are several mathematical models and information communication systems that allow to optimally assign the products on the warehouses' shelves (Tompkins et al., 2010). However, these models normally only consider the individual optimization of one of the several objectives involved in real operation. This frequently leads to the optimization of the selected objective while negatively affecting others objective functions to be optimized, which is not in accordance with the reality of the companies, in which all objectives and variables are important.

This paper presents a multiobjective optimization model with the aim of simultaneously minimizing the product handling costs and the required time to fulfill the orders in an industrial warehouse. The model helps the decision maker people to make the decisions about the products allocation, considering both critical objectives for the warehouse performance. Due to the complexity of the optimization process, a multiobjective genetic algorithm of the type NSGAI (NonDominated sorting Genetic Algorithm II) was developed to solve the allocation mathematical model.

The results produced by this genetic algorithm behave according to the model expectations, yielding not a single solution, but a set of possible solutions that simultaneously optimize both objective functions. The decision maker can select any of those solutions, based on its preferences, sure that each solution generates an optimal product allocation plan considering both functions for the warehouse.

## **2 Product Assignment in Warehouses**

Warehouses are facilities dedicated to the storage and care of products in organizations with the purpose of create a buffer to deal with the demand and supply variations (Zapata-Cortes et al., 2019; Ballesteros et al ., 2019). Storage is responsible for the generation of lots of cost in companies (Departamento Nacional de Planeación, 2018) due to the amount of economic, financial, personal and infrastructure resources required in such facilities for both the movement and storage of the goods (Chopra and Meindl, 2013). On the other hand, storage is also responsible for the service level perceived by customers, since it affect the lead time, the orders fulfillment and the quality and conditions of the products (Frazelle and Soho, 2010). Both the cost and service level are key factors for company's success (Chopra and Meindl, 2013). The costs optimization and the adequate service level are two of the main objectives pursued by warehouses' administrators, which can be achieved through multiple initiatives such as reducing the unnecessary distances and movements, improving the use of space, equipment, labor, accessibility to all items, among others.

These initiatives can be carried out through the use of information and communication technologies and information systems (Zapata et al., 2010), the appropriate facilities design (Arango et al., 2010) and the process optimization (De Koster et al., 2007). The use of technologies such as WMS (Warehouse Management System), barcode, radio frequency identification, among others, can increase the warehouse performance, reduce costs and improve the service levels of the storage operations (Zapata et al. , 2010).

An adequate design and the use of the right equipment and technology in the handling and storage processes, also impacts positively the performance of the warehouse. (Thomkins et al., 2010). The optimization refers to the adequate programming and allocation of people, equipment and materials, which can be done through mathematical procedures to determine the necessary resources, the optimal travel distances inside the warehouse for both the movement of people and equipment, the correct product location, the adequate quantities to be stored, among other activities (De Koster et al., 2007, Thomkins et al., 2010).

One alternative to improve the costs and response time in warehouse operations is to minimize the average travel distance, which means reducing the total distance to be travel in the warehouse (De Koster et al., 2007; Arango et al., 2010). This can be done by properly selecting the places where the goods should be located, which reduces the time and cost required to reach their storage positions (Zapata et a., 2019).

Many works and models found in the scientific literature consider the study of the single cost or operating time optimization (Sanei et al., 2011; Kovács, 2011; van Wijk et al., 2013; Zapata et al., 2019), while others researches perform joint optimization processes of several objectives (Ramtin & Pazour, 2015; Quintanilla et al., 2015; Hu et al. 2012), which allow to take decisions more in line with reality, where several objectives are important for the correct warehouse performance.

Multiobjective optimization differs from the conventional single-objective optimization since it does not generate a single solution, but a set of possible optimal solutions, from which the decision maker can select any according his preferences. This set of optimal solutions is known as the Pareto Frontier, where the improvement of one objective may result in the decrease of another or more objectives (Shenfield et al., 2007).

The Pareto frontier can be obtained with the use of conventional (classical) or through heuristic techniques (López et al., 2009). Classical methods present several disadvantages in the multi-objective optimization, since they require a high number of iterations to find the Pareto frontier, a prior knowledge of the problem domain, some of these methods are sensitive to the form or continuity of the Pareto frontier (López et al., 2009) and finding a satisfactory solution becomes increasingly complex as the number of objectives increase (Fonseca & Fleming, 1995). Heuristic methods allow to face the above-mentioned problems, finding good solutions (close to the optimal) in a reasonable processing time. Authors such as González (2013) and López et al., (2009) have analyzed the most used metaheuristic techniques to solve multiobjective optimization problems, among which are the NSGA and NSGA-II- (Nondominated Sorting Genetic Algorithm), SPEA and SPEA2 (Strength Pareto Evolutionary Algorithm), PAES (Pareto Archived Evolution Strategy), Multi-Objective Variable Environment Search (MO-VNS).

### **3 Multiobjective Model for Product Allocation in Warehouses**

The product allocation to the different positions in the warehouse can be done by optimizing the costs or time required to perform all the warehouse operations, which is

possible through the objective function presented in equation 1. This equation represents the costs minimization for the allocation problem where there are multiple products and several collecting/delivery points in the warehouse (Tompkins et al., 2010):

$$F1 = minimize \sum_{j=1}^n \sum_{k=1}^q \frac{T_j}{S_j} \sum_{i=1}^m p_i c_{ik} x_{jk}, \quad (1)$$

$x_{jk}$  is the binary decision variable, which takes the value of 1 if product  $j$  is assigned to position  $k$  or zero otherwise. The parameters description in the objective function is:

- $n$  is the number of products.
- $q$  is the number of storage positions.
- $m$  is the number of origin/delivery points in the warehouse.
- $T_j$  is the number of storage trips (input-output) for product  $j$ .
- $S_j$  is the number of storage positions required for product  $j$ .
- $p_i$  is the percentage of trips to go to  $i$  and return.
- $c_{ik}$  is the cost of travel from point  $i$  to the storage location  $k$ .

The objective function  $F1$  can be modified to optimize the time required to fulfill the orders. It can be done by replacing the costs ( $c_{ik}$ ) with the time required to perform this activity ( $t_{ik}$ : time required to go from  $i$  to the storage position  $k$ ). This generates the second objective function of the proposed model, presented in equation 2:

$$F2 = minimize \sum_{j=1}^n \sum_{k=1}^q \frac{T_j}{S_j} \sum_{i=1}^m p_i t_{ik} x_{jk}. \quad (2)$$

In this way, the model contemplates the costs and time for the expected travels distances for any product  $j$  from each origin point to its storage position  $k$ , which is represented by the term  $p_i d_{ik} x_{jk}$ .

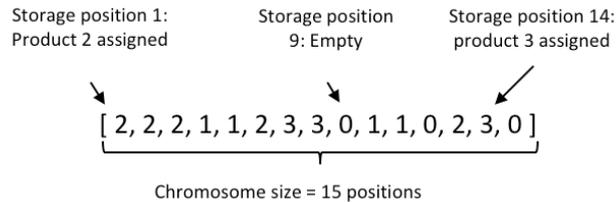
In addition, each objective function contemplates the movements intensity in terms of the storage positions with the term  $\frac{T_j}{S_j}$  required for each product  $J$ . With this, the total cost and time for the established operation time  $T_j$  are calculated.

Those objective functions are subject to the following constraints:

$$\sum_{j=1}^n x_{jk} = 1 \quad k = a, \dots, q, \quad (3)$$

$$\sum_{k=1}^q x_{jk} = S_j \quad j = a, \dots, n. \quad (4)$$

The probability that each item  $j$  travels from point  $i$  to each position  $k$  is the same for all products. Constraint 3 ensures that product  $j$  is assigned to position  $k$  only once. That is, only a product  $j$  can be assigned to a position  $k$ . Constraint 4 indicates that the quantity of products  $j$  assigned to positions  $k$  must be equal to the storage requirement (required spaces) for product  $j$ .



**Fig. 1.** Chromosome used for the product allocation problem.

#### 4 Multiobjective Genetic Algorithm for the Product Allocation Optimization

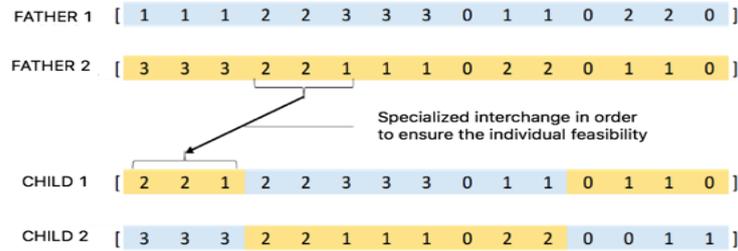
For the solution of the multiobjective model presented above, a genetic algorithm NSGAI (NonDominated Sorting Genetic Algorithm-II) was developed. This type of algorithm is one the most used multiobjective evolutionary optimization algorithms in the scientific literature (Zapata, 2016). Unlike many evolutionary algorithms, the NSGAI has the ability to work easily with the classification of individuals according to the non-dominance criterion. It also prevents the loss of good solutions since it constantly evaluates new and old individuals and contemplates the concept of diversity among them.

In the NSGAI algorithm, an initial population  $P_t = 0$  of size  $N$  is randomly created, which is organized according to the non-dominance of the individuals. This process is carried out through an iterative procedure in which different levels of non-dominance are determined, that is: Pareto Borders are obtained by separating the non-dominated solutions from the rest and then the non-dominated individuals are classified again with the non-dominance criterion. This process is carried out until all individual of the population is assigned to its non-dominance level. (Correa et al., 2008).

From this initial population, a new temporary population  $Q_0$  of size  $N$  is generated (Offspring population), through the selection, crossover and mutation operators of the genetic algorithm. These populations are combined to form a population  $R_t$  of size  $2N$  ( $P_t + Q_t$ ) which is ordered according to the non-dominance levels, thus ensuring elitism in the algorithm. From the different levels of non-dominance (Pareto borders) of  $R_t$ , a new population  $P_{t+1}$  is created including (accommodating) the individuals at the best levels of non-dominance (First Pareto borders) (Deb et al., 2002; Zapata, 2019).

Every individual in the NSGAI algorithm are represented as a vector of integers numbers (chromosome), in which each position  $i$  represents a storage position, as shown in Figure 1. To each position  $i$ , an integer number from zero to the number of different types of products is assigned, so the sum of positions with the same number is equal to the number of spaces in the warehouse required to storage such product. In this representation, a value 0 means that this space is empty.

The selection of individuals is made by tournament with size  $m$  ( $m = 5\%$  of the population size) which are randomly selected. These individuals are compared and selected according to the non-dominance and the dispersion criterion (Crowing Distance) (Correa et al., 2008).



**Fig. 2.** Crossover operator for product allocation problem.

**Table 1.** Solution individuals in the Pareto frontier.

Individual	Time	Cost	Individual	Time	Cost
1	40094	22549	7	39893	22600
2	40001	22577	8	40045	22557
3	39846	22729	9	39913	22591
4	40004	22573	10	39972	22578
5	39920	22579	11	39855	22723
6	40009	22563	12	39888	22722

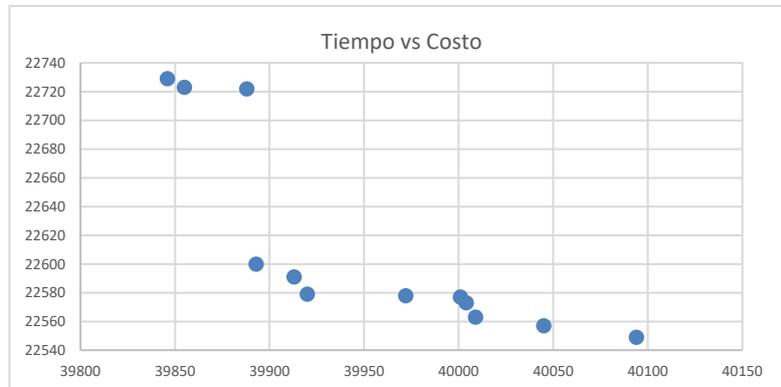
The crossover operator is performed by determining two points randomly selected, from which the parents' genetic information is exchanged to the children, as presented in Figure 2. To ensure the genetic feasibility of each child, a verify and adjust process is performed after the crossover.

The mutation operator is carried in two parts: in the first part, two positions are selected randomly and if the products in the positions are different, these are are exchanged. If the genes are the same, a new random selection is carried out until they are different. The second part is similar to the previous one, but this ensures that one of the positions randomly selected is empty, which gives the algorithm a greater search capability.

The conservation of the best individuals in the upcoming generations is implicit in the NSGAI algorithm, which ensures the elitism, as mentioned previously. The fitness function is evaluated by calculating the cost and time to allocate the products, as expressed in the objective functions equation presented above.

## 5 Discussion and Analysis of Results

The NSGAI algorithm was applied to solve the allocation problem in a company with 2500 storage positions and 3 products. The algorithm ran with a population of 100 individuals, 200 generations and a mutation percentage of 0.2. The number of optimal individuals in the Pareto frontier produced by the algorithm is 12 and their objective function values are presented in table 1.



**Fig. 2.** Pareto Frontier.

Those individual can be graphically presented in order to observe the Pareto frontier behavior in relation to the two objective functions, as presented in Figure 2.

From figure 2 it can be observed that both objective functions are contradictory, that is, the individual that generates the allocation plan with the best costs also produces the worst cost and conversely. In this way, all individuals in the Pareto frontier are optimal combination of the two objectives and the decision maker can select the one he considers that best suits its preferences and also can be sure that no solution is worse or better than any other.

Each of these individuals generates a product allocation plan in the warehouse, indicating what type of product should be storage in each space, as it is defined by the chromosome representation in the genetic algorithm. In this way, it is easy for the decision maker to convert the answer of the algorithm(the individuals) into the warehouse allocation plan. Because to the size of the table that represent each individual solution (a vector with 2500 positions), their presentation is omitted in this paper.

## 6 Conclusions

This work presented a product allocation model that contemplates the simultaneous optimization of two fundamental objectives for the correct operation and performance in warehouses, which are the costs and the time to fulfil the orders and make the operations. In this way, for the operative plans, managers can find a relationship between these two objective functions that ensures a combination that optimizes both criteria at the same time.

Due to the complexity of the proposed model and the disadvantages of the classic methods to solve multiobjective problems, a genetic algorithm NSGAI2 was developed, which used an easy-to-understand representation that facilitates the algorithm operation and the conversion of the solutions into the product allocation plan in warehouses. The algorithm ran according to it was expected, generating the Pareto frontier of optimal individuals, from which the decision maker can select any solutions, with the assurance of using an optimal allocation plan in relation to the cost and service time in the warehouse.

As future research work, it is recommended to integrate this model into an information system that not only allows the development of optimal allocation plans, but also provide guidance to workers about the location of the products. In addition, as future research lines, it is suggested to analyze other objective functions to optimize, such as the required area for the allocation plans in the warehouse and other variables, such as costs and times of order preparation.

## References

1. Arango-Serna, M.D., Zapata-Cortes, J.A., Pemberthy, J.I.: Reestructuración del layout de la zona de picking en una bodega industrial. *Revista Ingeniería Universidad de los Andes*, 32, pp. 54–61 (2010)
2. Ballesteros-Riveros, F.A., Arango-Serna, M.D., Adarme-Jaimes, W., Zapata-Cortes, J.A.: Storage allocation optimization model in a Colombian company. *DYNA*, 86(209), pp. 255–260, (2019)
3. Chopra, S., Meindl, P.: *Administración de la cadena de suministro*. Pearson Education, Mexico (2013)
4. Correa, C.A., Bolaños, R.A., Molina, A.: Algoritmo multiobjetivo NSGA-II aplicado al problema de la mochila. *Scientia et Technica*, 14(39), pp. 206–211 (2008)
5. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.A.: Fast y elitist multiobjective genetic algorithm: NSGA-II. In *Proceeding IEEE Transactions on Evolutionary Computation*, 6(2), (2002)
6. De Koster, R., Le-Duc, T., Roodbergen, K.J.: Design and control of warehouse order picking: a literature review. *Eur. J. Oper. Res.*, 182, pp. 481–501 (2007)
7. Departamento Nacional de Planeación: *Encuesta Nacional Logística 2018* (2018)
8. Fonseca, C.M., Fleming, P.J.: An overview of evolutionary algorithms in multiobjective optimization. *Evolutionary Computing*, 3(1), pp. 1–16 (1995)
9. Frazelle, E., Soho, R.: *World-class warehousing and material handling*. McGraw-Hill Education (2016)
10. González, D.L.: *Metaheurísticas, optimización multiobjetivo y paralelismo para descubrir motifs en secuencias de AND*. Universidad de Extremadura (2013)
11. Hu, W., Wang, Y., Zheng, J.: Research on warehouse allocation problem based on the artificial bee colony inspired particle swarm optimization (ABC-PSO) algorithm. *International Symposium on Computational Intelligence and Design*, 1, pp. 173–176 (2012)
12. Kovács, A.: Optimizing the storage assignment in a warehouse served by milkrun logistics. *International Journal of Production Economics*, 133(1), pp. 312–318 (2011)
13. López, A., Zapotecas, S., Coelho, L.C.: An introduction to multiobjective optimization techniques, In *Evolutionary Multiobjective Optimization: Theoretical Advances and Applications*, pp. 7–32. Springer (2009)
14. Quintanilla, S., Pérez, Á., Ballestín, F., Lino, P.: Heuristic algorithms for a storage location assignment problem in a chaotic warehouse. *Engineering* (2015)
15. Ramtin, F., Pazour, J.A.: Product allocation problem for an AS/RS with multiple in-the-aisle pick positions. In *IIE Transactions*, 47(12), pp. 1379–1396 (2015)
16. Sanei, O., Nasiri, V., Marjani, M.R., Moattar, H.S.M.: A heuristic algorithm for the warehouse space assignment problem considering operational constraints: with application in a case study. pp. 258–264 (2011)
17. Shenfield, A., Fleming, P.J., Alkarouri, M.: Computational steering of a multi-objective evolutionary algorithm for engineering design. *Engineering Applications of Artificial Intelligence*, 20, pp. 1047–1057 (2007)

18. Tompkins, J.A., White, J.A., Bozer, Y.A., Tanchoco, J.M.A.: Facilities planning. Wiley, (2010)
19. Van Wijk, A.C.C., Adan, I.J.B.F., van Houtum, G.J.: Optimal allocation policy for a multi-location inventory system with a quick response warehouse. *Operations Research Letters*, 41(3), pp. 305–310 (2013)
20. Zapata-Cortes, J.A., Arango-Serna, M.D., Serna-Urán, C.A., Adarme-Jaimes, W.: Mathematical model for product allocation in warehouses. In: García-Alcaraz J., Sánchez-Ramírez C., Avelar-Sosa L., Alor-Hernández G. (eds) *Techniques, tools and methodologies applied to global supply chain ecosystems*. Intelligent Systems Reference Library, 166, Springer, Cham (2020)
21. Zapata-Cortes, J.A., Arango-Serna, M.D., Adarme-Jaimes, W.: Herramientas tecnológicas al servicio de la gestión empresarial. *Avances en Sistemas de Información*, 7(3), pp. 87–102 (2010)
22. Zapata-Cortes, J.A.: Optimización de la distribución de mercancías utilizando un modelo genético multiobjetivo de inventario colaborativo de m proveedores con clientes. Universidad Nacional de Colombia, (2016)