# Radiant Flux Analysis of a System based in Imaging Fresnel Lens and Plastic Optical Fiber with Fiber Lenses 

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#### Abstract

Imaging Fresnel has been extensively studied to focus light in an effective and low-cost way; however, these lenses show low tolerances in alignment when they are coupled with plastic optical fibers (POFs). The principal factor is the high focal point movements produced by slight changes in the incidence angle of the source rays. In order to study these movements and its effects over the coupling between Fresnel lens and POFs, a mathematical analysis using Geometrical Optics and simulations in Zemax OpticStudio ${ }^{\circledR}$ were performed. The results show that the asymmetry of the numeric aperture of the Fresnel lens has higher sensitivity to low incidence angles and the coupling of the focused light to POFs with different fiber lenses at the input have a better performance using a plane face end. The results confirm the low tolerance, $<1^{\circ}$, to source angular misalignments and suggest the use of a secondary optical element.


Keywords: Imaging Fresnel lens; fiber lenses; plastic optical fiber; radiometry; geometrical optics.

## 1 Introduction

Since its invention in 1822 by Augustin Fresnel, the Fresnel lenses have been used with different purposes as lighthouses, imaging systems and light concentration; indeed, focus light, in particular Solar light, using Fresnel lens is an effective way to make full use of sunlight (Leutz \& Suzuki, 2001; Xie, Dai, Wang, \& Sumathy, 2011).

Indoor illumination by sunlight concentration with Fresnel lenses is one of the uses of imaging Fresnel lenses which has grown its implementation due to the importance of reduce the energy consumption in modern societies (Xie et al., 2011).

Imaging Fresnel lenses offer flexibility in optical design thanks to its high manufacture error tolerance, light-weight, small volume and low cost in contrast with conventional lenses; but the Imaging Fresnel lenses are prone to focal point movements due to changes in the incidence angle of the source rays, this leads, in the case of sunlight collection, to the need of high-precision Solar tracking for focal foreshortening and longitudinal focal movements (Leutz \& Suzuki, 2001).

This work shows a mathematical analysis of the variations presented in the focal point position when the source rays arrive parallel between them but form an angle from the optical axis. Considering the afore mentioned analysis, a numeric analysis for compute the changes produced in the output power in a system composed by an Imaging Fresnel lens and a Plastic Optical Fiber (POF) with different fiber lenses in its input was performed, the light source has parallel rays arriving with angles from $0^{\circ}$ to $4^{\circ}$ from the system optical axis.

## 2 Materials and Methods

The Fresnel lens are essentially chains of prisms and can be studied using Geometrical Optics considering the phenomena of reflection and refraction in each prism (Leutz \& Suzuki, 2001).


Fig. 1. Imaging Fresnel lens with grooves facing inward and source rays arriving parallel to optical axis, $\theta_{0}$, and other arriving with an angle $\phi_{0}$. The marginal rays for the 2 inputs generate 2 different output cones delimited by the marginal rays in each case.

For the coupling analysis between the Imaging Fresnel lens and the POF it is necessary know the output generated by a Fresnel lens. According to this, Snell's law was used (Leutz \& Suzuki, 2001):

$$
\begin{equation*}
n \sin \theta=n^{\prime} \sin \theta^{\prime} \tag{1}
\end{equation*}
$$

And the relations for the angles $\alpha$ and $\beta$, which are (Leutz \& Suzuki, 2001):

$$
\begin{gather*}
\alpha=\tan ^{-1} R / f=\theta_{\text {output }}-\beta,  \tag{2}\\
\beta=\tan ^{-1}\left(\frac{R}{n^{\prime \sqrt{R^{2}+f^{2}}}-f}\right) . \tag{3}
\end{gather*}
$$

Where the angle $\alpha$ is the semi-angle of the output cone, $\beta$ is the angle of the prism, R is the radial length of the Fresnel lens, f its focal length, n ' represents the refractive index of the Fresnel material and $\theta_{\text {output }}$ is the angle of the marginal ray with the
normal of the prism (Figure 1). The equation for the $\beta$ angle is only valid for Fresnel lens with grooves facing inward (Leutz \& Suzuki, 2001).

With the aforementioned equations and applying the Snell's law in every interface in the marginal prism, the relation between the radial length and the focal length, when the source rays arrive with an angle $\theta_{0}=0^{\circ}$ and the output ray has a marginal ray with an angle $\theta 1$, can be expressed as follow:

$$
\begin{equation*}
R / f=\tan \left(\theta_{1}-\beta\right)=\tan \left[\sin ^{-1}\left(n^{\prime} \sin \beta\right)-\beta\right] . \tag{4}
\end{equation*}
$$

For this case, the output cone is radially symmetric with the optical axis:

$$
\begin{equation*}
\alpha=\alpha^{\prime} \tag{5}
\end{equation*}
$$

Now, for the case when the source rays arrive with an angle $\phi_{0}$ from the optical axis and considering $\beta$ as a constant physical parameter of the marginal prism, the relation between $R^{\prime}$ and $f^{\prime}$, shown in Figure 1, is defined as:

$$
\begin{equation*}
\frac{R^{\prime}}{f^{\prime}}=\tan \left[-\beta+\sin ^{-1}\left(n^{\prime} \sin \left[\beta-\sin ^{-1}\left(\frac{\sin \phi_{0}}{n^{\prime}}\right)\right]\right)\right]=\tan \alpha_{1} \tag{6}
\end{equation*}
$$

And, as this case is not radially symmetrical, $\alpha_{1} \neq \alpha_{2}$ the relation between $R^{\prime \prime}$ and $f^{\prime}$, shown in Figure 1, is:

$$
\begin{equation*}
\frac{R^{\prime \prime}}{f^{\prime}}=\tan \left[-\beta+\sin ^{-1}\left(n^{\prime} \sin \left[\beta+\sin ^{-1}\left(\frac{\sin \phi_{0}}{n^{\prime}}\right)\right]\right)\right]=\tan \alpha_{2} \tag{7}
\end{equation*}
$$

For a quantitative analysis of the equations obtained, a Fresnel lens with a radius of 14.29 cm , a focal length of 16.51 cm and a refractive index of 1.491 was considered. This lens was used to see the differences angles, $\alpha_{1}$ and $\alpha_{2}$, generated by the tilt angle in the incidence light source, considering angles $\phi_{0}=0.0^{\circ}: 5.0^{\circ}$ with steps of $0.1^{\circ}$. The results of this analysis are shown in the next section (Figure 4) and present the lineal increase of the asymmetry in the output cone of the Fresnel lens.

A conventional POF, which has a Polymethyl methacrylate core ( $n=1.491$ ) and with a fluorinated polymer cladding ( $n=1.418$ ), has an acceptance semi-angle $\sim 27.23^{\circ}$, which represents a Fresnel lens as the one used for the above analysis will not be able to couple with a POF, for this reason it is necessary look for alternative ways to increase the coupling between the Fresnel lens and the POF.

Considering the acceptance angle of a POF can be modified using fiber lenses (Bescherer, Munzke, Reich, \& Loock, 2013), Zemax OpticStudio ${ }^{\circledR}$ was used to simulate an optical system composed by a D65 light source, a Fresnel lens and 10 cm of POF (Figure 2).

Different input shapes were analyzed at the input POF, being the 4 fiber lenses shown in Figure 3 the shapes with the best results. The parameters for the fiber lenses were taken from (Viera-González et al., 2013).

During the simulations the radiant flux was measured tilting the light source from $0^{\circ}$ to $5^{\circ}$, in steps of $0.1^{\circ}$, for every fiber lens; the output power and coupling was analyzed. The output of a POF with planar input when the incidence angle of the source rays is $0^{\circ}$ was taken as the base output power (100\%).


Fig. 2. Optical system design in Zemax OpticStudio ${ }^{\circledR}$ : the Fresnel lens has an output semi-angle of $40.87^{\circ}$, the POF has an acceptance semi-angle of $27.23^{\circ}$.


Fig. 3. Fiber lenses. a) Conventional POF with planar input. b) POF with concave fiber lens, the curvature corresponds to a parabolic surface with curvature radius of 10. c) POF with a conic cavity in the input, the aperture semi-angle of the cone is $17^{\circ}$. d) Fiber lens with conic form, the semi-angle of the cone is the same as the one of the conic cavity.

## 3 Results

The Figure 4 shows the results of the mathematical analysis, where $\Delta \alpha=\left|\alpha_{1}-\alpha_{2}\right|$, where is proved the focal movements due to angular misalignment between light source and Fresnel lens; these changes in the focal length also reflect a change in the output cone, breaking the symmetry presented in the case when the light arrives totally perpendicular to the Fresnel lens.


Fig. 4. The graph shows a lineal growth of the asymmetry in the output cone in the range between $\phi_{0}=0.1^{\circ}: 0.6^{\circ}$, out of this range it is not possible determine te output angle in one of the marginal prisms, because for this cases the total internal reflection is presented.

For the particular case of the Fresnel lens used for the mathematical analysis, when the incidence angle of the light is higher than $\phi_{0}=0.6^{\circ}$, the marginal prisms presents total internal reflection that generated losses.

The results of the simulation mentioned in the last section are summarized in Fig. 5, this graph shows the decrement of the system efficiency for the angular misalignment between the light source and the Fresnel lens.


Fig. 5. Plane and concave FOP faces have better performance. The plane, concave and conic cavity fiber lenses show similar performance and have, in the theoretical critical tilt of the Fresnel lens $\left(0.7^{\circ}\right)$, efficiency around $80-90 \%$.

## 4 Discussion

The mathematical analysis made for the asymmetry generated by angular misalignments between the light source and the Fresnel lens shows low tolerance ( $<$ $1^{\circ}$ ), because the asymmetry is increased; also, with the mathematical relations found it is possible calculate the changes produced in the focus position. This low tolerance represents a problem for applications with sources in movement, as solar illumination systems.

The results obtained in (Viera-González et al., 2013) could not be applied in fiber inputs to increase the angular coupling between POF and Fresnel lens; then, for achieve a higher coupling, without use tracking systems, will be necessary the use of a secondary optical element for reduce the angle of the light that arrives to the POF and, at the same time, generate a uniform and radially symmetrical spot which couples with the fiber.

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